PARAMETRIC METHOD OF CALCULATING A TRANSIENT LAMINAR BOUNDARY LAYER

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A universal equation is derived for a transient laminar boundary layer in an incompressible fluid and is solved by numerical integration for certain values of the chosen parameters.

The Loitsyanskii method [1-3] with a "universal" equation of a laminar boundary layer offers certain definite advantages over other approximate methods and can also be applied to a transient flow. A replacement of both the longitudinal and the time coordinate by a sequence of parameters representing the effect of velocity variations in the outer stream and, in integral form, the flow history in the boundary layer will make it possible to integrate the universal equation once in advance for all cases. The results of this integration alone are of interest, because they reveal various trends in the development of a transient boundary layer and, furthermore, they yield a set of profiles which can be used for solving specific problems with given velocity distributions U(x, t) at the outer edge of the boundary layer.

We will note that Duric [4] has extended the Loitsyanskii method to derive a universal equation for a transient boundary layer and solved it by an expansion in series. This equation is valid only under the assumption, however, that the velocity at the outer edge of the boundary layer U(x, t) can be expressed as the product of two functions: a function of the longitudinal coordinate only and a function of time only. In our case there is no such restriction and the quantity U(x, t) can be any analytic function of both the longitudinal coordinate and time.

1. We write the equation of a two-dimensional transient laminar boundary layer in an incompressible fluid as follows:

$$\frac{\partial^2\psi}{\partial t\partial y}+\frac{\partial\psi}{\partial y}\cdot\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\cdot\frac{\partial^2\psi}{\partial y^2}=\dot{U}+UU'+\nu\frac{\partial^3\psi}{\partial y^3} \tag{1}$$

with the boundary conditions;

$$\psi=\frac{\partial\psi}{\partial y}=0 \quad \text{at} \quad y=0,$$

$$\frac{\partial\psi}{\partial y}\to U(x,\ t) \quad \text{at} \quad y\to\infty,$$

$$\frac{\partial\psi}{\partial y}=u_1(x,y) \quad \text{at} \quad t=t_0, \quad \frac{\partial\psi}{\partial y}=u_0(t,\ y) \quad \text{at} \quad x=x_0. \tag{2}$$

Here x, y, t are the longitudinal coordinate, the transverse coordinate, and time, respectively; U(x, t) is the velocity at the outer edge of the boundary layer; ν is the kinematic viscosity; and ψ(x, y, t) is a flow function which, as usual, satisfies the equalities

$$u=\frac{\partial\psi}{\partial y};\quad v=-\frac{\partial\psi}{\partial x}. \tag{3}$$

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A prime and a dot above a symbol will denote a partial derivative with respect to x and t, respectively.

The last row of boundary conditions in (2) signifies that the solution to Eq. (1) is sought for $t > t_0$ and $x > x_0$, under the assumption that the flow function ψ is known over the entire boundary layer at the instant $t = t_0$ and also across the transverse section at $x = x_0$ during all times t. At $t = t_0$ the flow function ψ can be expressed in various ways, depending on the flow mode at a given instant of time.

We now introduce new variables

$$x = x; \quad t = t; \quad \eta = \frac{By}{h(x, t)}; \quad \varphi(x, \eta, t) = \frac{B\psi(x, y, t)}{U(x, t) h(x, t)}, \tag{4}$$

where h(x, t) is some characteristic linear scale function for the transverse coordinate in the boundary layer and B is the normalizing coordinate.

In this case differentiation with respect to the old coordinates x, y, t is replaced by the following differentiation:

$$\frac{\partial}{\partial x} = \left(\frac{\partial}{\partial x}\right) + \frac{\partial}{\partial \eta} \cdot \frac{\partial \eta}{\partial h} \cdot \frac{\partial h}{\partial x} = \left(\frac{\partial}{\partial x}\right) - \eta \frac{h'}{h} \cdot \frac{\partial}{\partial \eta},$$

$$\frac{\partial}{\partial t} = \left(\frac{\partial}{\partial t}\right) + \frac{\partial}{\partial \eta} \cdot \frac{\partial \eta}{\partial h} \cdot \frac{\partial h}{\partial t} = \left(\frac{\partial}{\partial t}\right) - \eta \frac{\dot{h}}{h} \cdot \frac{\partial}{\partial \eta},$$

$$\frac{\partial}{\partial y} = \frac{B}{h} \cdot \frac{\partial}{\partial \eta}; \quad \frac{\partial^2}{\partial y^2} = \frac{B^2}{h^2} \cdot \frac{\partial^2}{\partial \eta^2}; \quad \frac{\partial^3}{\partial y^3} = \frac{B^3}{h^3} \cdot \frac{\partial^3}{\partial \eta^3}. \tag{5}$$

With the aid of (4) and (5), letting $z = h^2/\nu$, we reduce Eq. (1) to the following form:

$$B^2 \frac{\partial^3 \varphi}{\partial \eta^3} + U'z\left[\varphi \frac{\partial^2 \varphi}{\partial \eta^2} - \left(\frac{\partial \varphi}{\partial \eta}\right)^2 + 1\right] + \frac{\dot{U}z}{U}\left(1 - \frac{\partial \varphi}{\partial \eta}\right) + \frac{Uz'}{2}\varphi \frac{\partial^2 \varphi}{\partial \eta^2}$$
$$+ \frac{\dot{z}}{2}\eta \frac{\partial^2 \varphi}{\partial \eta^2} - z\frac{\partial^2 \varphi}{\partial t \partial \eta} + Uz\left(\frac{\partial \varphi}{\partial x} \cdot \frac{\partial^2 \varphi}{\partial \eta^2} - \frac{\partial \varphi}{\partial \eta} \cdot \frac{\partial^2 \varphi}{\partial x \partial \eta}\right) = 0 \tag{6}$$

with the boundary conditions

$$\varphi = \frac{\partial \varphi}{\partial \eta} = 0 \quad \text{at} \quad \eta = 0,$$

$$\frac{\partial \varphi}{\partial \eta} \to 1 \quad \text{at} \quad \eta \to \infty. \tag{7}$$

The boundary conditions in the last row of (2) can be used further whenever a specific problem is to be solved; they should be omitted in the derivation of the universal equation.

2. With the multiparameter set of velocity profiles across sections of the boundary layer given as

$$\bar{u} = \frac{u}{U}(\eta, f_{kn}), \text{ where } k = 0, 1, 2, \ldots, n = 0, 1, 2, \ldots,$$

or, for the flow function

$$\psi = \frac{1}{B} Uh\varphi(\eta, f_{kn}) \tag{8}$$

and with function U(x, t) assumed analytic, we introduce a system of parameters

$$f_{kn} = U^{k-1} \frac{\partial^{k+n} U}{\partial x^k \partial t^n} z^{k+n} \quad (k = 0, 1, 2, \ldots; n = 0, 1, 2, \ldots). \tag{9}$$

From expression (9) follows

$$f_{10} = U'z; \quad f_{01} = \frac{\dot{U}}{U} z. \tag{10}$$

This chosen parameter structure conveys, on the one hand, the effect of velocity variations (with time and along the x coordinate) in the outer stream and, on the other hand, the flow history in the boundary layer expressed in integral form. For an arbitrary function U(x, t) each of the parameters will act as the independent variable in lieu of x and t. The parameter $f_{00} = 1 = \text{const}$ is excluded from consideration. For n = 0 the parameters (9) become $f_{kn} = U^{k-1}(\partial^k U/\partial x^k)z^k$, identical with the parameters for a steady-state boundary layer [1, 2, 3].

In order to transform Eq. (6), we first calculate the derivatives in it. Thus,

$$\frac{\partial\varphi}{\partial x} = \sum_{k,n=0}^{\infty} \frac{\partial\varphi}{\partial f_{kn}} \cdot \frac{\partial f_{kn}}{\partial x},$$

$$\frac{\partial^2\varphi}{\partial x\partial\eta} = \sum_{k,n=0}^{\infty} \frac{\partial^2\varphi}{\partial f_{kn}\partial\eta} \cdot \frac{\partial f_{kn}}{\partial x}, \tag{11}$$

$$\frac{\partial^2\varphi}{\partial t\partial\eta} = \sum_{k,n=0}^{\infty} \frac{\partial^2\varphi}{\partial f_{kn}\partial\eta} \cdot \frac{\partial f_{kn}}{\partial t},$$

where the derivatives of the parameters with respect to x and t are found by a direct differentiation of expression (9). We have

$$\frac{\partial f_{kn}}{\partial x} = (k-1)\,U^{k-2}U'\,\frac{\partial^{k+n}U}{\partial x^k\partial t^n}\,z^{k+n} + U^{k-1}\,\frac{\partial^{k+1+n}U}{\partial x^{k+1}\partial t^n}\,z^{k+n} + U^{k-1}\,\frac{\partial^{k+n}U}{\partial x^k\partial t^n}\,(k+n)\,z^{k+n-1}z'$$

$$= \frac{1}{zU}\left[(k-1)\,f_{10}f_{kn} + (k+n)\,f_{kn}Uz' + f_{k+1,n}\right] \tag{12}$$

and

$$\frac{\partial f_{kn}}{\partial t} = (k-1)\,U^{k-2}\dot{U}\,\frac{\partial^{k+n}U}{\partial x^k\partial t^n}\,z^{k+n} + U^{k-1}\,\frac{\partial^{k+n+1}U}{\partial x^k\partial t^{n+1}}\,z^{k+n} + (k+n)\,U^{k-1}\,\frac{\partial^{k+n}U}{\partial x^k\partial t^n}\,z^{k+n-1}\dot{z}$$

$$= \frac{1}{z}\left[(k-1)\,f_{01}f_{kn} + (k+n)\,f_{kn}\dot{z} + f_{k,n+1}\right]. \tag{13}$$

Using formulas (11)-(13), we transform Eq. (6) into

$$\Phi(\eta, f_{kn}) + D(\eta, f_{kn})\,\dot{z} + E(\eta, f_{kn})\,Uz' = 0, \tag{14}$$

where

$$\Phi(\eta, f_{kn}) = B^2\frac{\partial^3\varphi}{\partial\eta^3} + f_{10}\left[\varphi\frac{\partial^2\varphi}{\partial\eta^2} - \left(\frac{\partial\varphi}{\partial\eta}\right)^2 + 1\right]$$

$$+ f_{01}\left(1 - \frac{\partial\varphi}{\partial\eta}\right) - \sum_{k,n=0}^{\infty}\Bigg\{\left[(k-1)\,f_{01}f_{kn} + f_{k,n+1}\right]\frac{\partial^2\varphi}{\partial f_{kn}\partial\eta}$$

$$- \left[(k-1)\,f_{10}f_{kn} + f_{k+1,n}\right]\left(\frac{\partial\varphi}{\partial\eta}\cdot\frac{\partial^2\varphi}{\partial f_{kn}\partial\eta} - \frac{\partial\varphi}{\partial f_{kn}}\cdot\frac{\partial^2\varphi}{\partial\eta^2}\right)\Bigg\}, \tag{15}$$

$$D(\eta, f_{kn}) = \frac{\eta}{2}\cdot\frac{\partial^2\varphi}{\partial\eta^2} - \sum_{k,n=0}^{\infty}(k+n)\,f_{kn}\,\frac{\partial^2\varphi}{\partial f_{kn}\partial\eta},$$

$$E(\eta, f_{kn}) = \frac{\varphi}{2}\cdot\frac{\partial^2\varphi}{\partial\eta^2} - \sum_{k,n=0}^{\infty}(k+n)\,f_{kn}\left(\frac{\partial\varphi}{\partial\eta}\cdot\frac{\partial^2\varphi}{\partial f_{kn}\partial\eta} - \frac{\partial\varphi}{\partial f_{kn}}\cdot\frac{\partial^2\varphi}{\partial\eta^2}\right).$$

In order to make Eq. (14) universal, it is necessary to express the factors $\dot{z}$ and Uz', which are functions of coordinate x and time t, in terms of quantities which are explicit functions of the parameters f_{kn}. In other words, the following equalities

$$\dot{z} = F(f_{kn}), \quad Uz' = T(f_{kn}). \tag{16}$$

must hold true.

3. Functions F and T will be determined on the basis of momentum and energy equations, which for a transient boundary layer may be written as

$$\frac{\partial}{\partial t}(U\delta^*) + \frac{\partial}{\partial x}(U^2\delta^{**}) + UU'\delta^* - \frac{\tau_w}{\rho} = 0 \tag{17}$$

and

$$U^2\dot{\delta}^* + \frac{\partial}{\partial t}(U^2\delta^{**}) + U^3(\delta_1^{**})' + 3U^2U'\delta_1^{**} - 2\nu U^2 e = 0, \tag{18}$$

with the following designations:

$$\delta^*(x,\ t) = \int_0^\infty \left(1 - \frac{u}{U}\right) dy, \tag{19}$$

$$\delta^{**}(x,\ t) = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy, \tag{20}$$

$$\tau_w(x,\ t) = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0}, \tag{21}$$

$$\delta_1^{**}(x,\ t) = \int_0^\infty \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right) dy, \tag{22}$$

$$e(x,\ t) = \int_0^\infty \left[\frac{\partial\left(\frac{u}{U}\right)}{\partial y}\right]^2 dy. \tag{23}$$

Before determining the functions F and T, one must somehow choose the linear scale function h(x, t) in expressions (4). We will impose on h(x, t) the following requirements†:

$$\frac{\partial}{\partial x}(U^2h) = \frac{\partial}{\partial t}(U\delta^*) + \frac{\partial}{\partial x}(U^2\delta^{**}) \tag{24}$$

and

$$\frac{\partial}{\partial t}(U^2h) = \frac{\partial}{\partial t}(U^2\delta^{**}) + U^3(\delta_1^{**})' + 3U^2U'\delta_1^{**} - 2\nu U^2 e. \tag{25}$$

Considering a steady flow, equality (24) yields the conventional expression for the scale function h = δ^{**}, while relation (25) becomes transformed into the energy equation for the steady case. This choice of a scale in accordance with equalities (24) and (25) leads to some further simplifications and – what is especially important – allows the computer calculation of the universal equation to be programmed for an easy transition to the limiting case of steady flow.

The momentum equation (17) and the energy equation (18), with (24) and (25) taken into account, become

$$\frac{\partial}{\partial x}(U^2h) + UU'\delta^* - \frac{\tau_w}{\rho} = 0 \tag{26}$$

and

$$U^2\dot{\delta}^* + \frac{\partial}{\partial t}(U^2h) = 0. \tag{27}$$

Equation (26) will now be transformed as follows. Resolving the derivative with respect to x and substituting for τ_W its value according to (21), we have

$$2UU'h + U^2h' + UU'\delta^* - \nu\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0.$$

† An analogous method was applied by V. V. Bogdanova in her analysis of a three-dimensional boundary layer.

Multiplying both sides of this equation by $h/\nu U$, we obtain

$$2U'\frac{h^2}{\nu}+\frac{U}{2}\left(\frac{h^2}{\nu}\right)'+U'\delta^*\frac{h}{\nu}-\left.\frac{\partial\left(\frac{u}{U}\right)}{\partial\left(\frac{y}{h}\right)}\right|_{y=0}=0$$

or

$$2U'z+\frac{U}{2}z'+U'zH^*-\zeta=0, \tag{28}$$

where

$$H^*=\frac{\delta^*}{h}, \quad \zeta=\left.\frac{\partial\left(\frac{u}{U}\right)}{\partial\left(\frac{y}{h}\right)}\right|_{y=0}. \tag{29}$$

On the basis of (19), (3), and (4), the quantities H^* and ζ can be expressed as

$$H^*=\int_0^\infty\left(1-\frac{u}{U}\right)d\left(\frac{y}{h}\right)=\frac{1}{B}\int_0^\infty\left(1-\frac{\partial\varphi}{\partial\eta}\right)d\eta \tag{30}$$

and

$$\zeta=\left[\frac{\partial\left(\frac{u}{U}\right)}{\partial\left(\frac{y}{h}\right)}\right]_{y=0}=B\left.\frac{\partial^2\varphi}{\partial\eta^2}\right|_{\eta=0}. \tag{31}$$

It is evident here that $H^* = H^*(f_{kn})$ and $\zeta = \zeta(f_{kn})$ are functions of the parameters f_{kn} ($k = 0, 1, 2, \ldots, n = 0, 1, 2, \ldots$) only.

It follows from (28) and the first equality in (10) that also the quantity

$$Uz'=2[\zeta-f_{10}(2+H^*)]=F(f_{kn}) \tag{32}$$

is a function of these parameters only. We note that this expression is of the same form as the analogous equality for a steady boundary layer.

With the aid of the first equality in (29) and after resolving the derivatives with respect to t, we transform Eq. (27) into

$$U^2\dot{h}H^*+U^2h\sum_{k,n=0}^{\infty}\frac{\partial H^*}{\partial f_{kn}}\dot{f}_{kn}+2U\dot{U}h+U^2\dot{h}=0.$$

Multiplying both sides of this equation by $h/\nu U^2$ will yield

$$\frac{h\dot{h}}{\nu}H^*+\frac{h^2}{\nu}\sum_{k,n=0}^{\infty}\frac{\partial H^*}{\partial f_{kn}}\dot{f}_{kn}+2\frac{\dot{U}}{U}\cdot\frac{h^2}{\nu}+\frac{h\dot{h}}{\nu}=0$$

or, taking into consideration the second equality in (10),

$$\frac{H^*}{2}\dot{z}+\sum_{k,n=0}^{\infty}\frac{\partial H^*}{\partial f_{kn}}z\dot{f}_{kn}+2f_{01}+\frac{\dot{z}}{2}=0.$$

Finally, using equality (13), we have

$$\dot{z}=-2\frac{2f_{01}+\sum\limits_{k,n=0}^{\infty}[(k-1)f_{01}f_{kn}+f_{k,n+1}]\dfrac{\partial H^*}{\partial f_{kn}}}{H^*+1+2\sum\limits_{k,n=0}^{\infty}\dfrac{\partial H^*}{\partial f_{kn}}(k+n)f_{kn}}=T(f_{kn}). \tag{33}$$

The quantity $T(f_{kn})$, just as $F(f_{kn})$, is a function of the parameters f_{kn} only.

In this way, we have proved the existence of equalities (16). It is noteworthy that Eqs. (32) and (33) are not compatible. This inexactness is permissible, however, as far as the method is approximate.

According to (15), (32), and (33), Eq. (14) becomes

$$B^2 \frac{\partial^3 \varphi}{\partial \eta^3} + \left(\frac{F}{2} + f_{10} \right) \varphi \frac{\partial^2 \varphi}{\partial \eta^2} + f_{10} \left[1 - \left(\frac{\partial \varphi}{\partial \eta} \right)^2 \right] + \frac{T}{2} \eta \frac{\partial^2 \varphi}{\partial \eta^2}$$

$$+ f_{01} \left(1 - \frac{\partial \varphi}{\partial \eta} \right) = \sum_{k,n=0}^{\infty} \left\{ \left[(k+n) F f_{kn} + (k-1) f_{10} f_{kn} + f_{k+1,n} \right] \right.$$

$$\left. \times \left(\frac{\partial \varphi}{\partial \eta} \cdot \frac{\partial^2 \varphi}{\partial f_{kn} \partial \eta} - \frac{\partial \varphi}{\partial f_{kn}} \cdot \frac{\partial^2 \varphi}{\partial \eta^2} \right) + \left[(k+n) T f_{kn} + (k-1) f_{01} f_{kn} + f_{k,n+1} \right] \frac{\partial^2 \varphi}{\partial f_{kn} \partial \eta} \right\}. \tag{34}$$

Neither the velocity at the edge of the boundary layer nor its derivatives appear in the last equation and, therefore, it may be called a universal equation. The boundary conditions are also universal, i.e., they are

$$\varphi = \frac{\partial \varphi}{\partial \eta} = 0 \quad \text{at} \quad \eta = 0; \quad \frac{\partial \varphi}{\partial \eta} \to 1 \quad \text{at } \eta \to \infty,$$

$$\varphi = \varphi_0(\eta) \quad \text{at} \quad f_{kn} = 0 \quad (k = 0, 1, 2, \ldots; \quad n = 0, 1, 2, \ldots). \tag{35}$$

Function $\varphi_0(\eta)$ is the Blasius solution for a steady boundary layer at a plate. If all f_{kn} parameters are assumed equal to zero, we have the equation

$$\frac{\partial^3 \varphi_0}{\partial \eta^3} + \frac{\zeta_0}{B^2} \varphi_0 \frac{\partial^2 \varphi_0}{\partial \eta^2} = 0,$$

which becomes the well-known Blasius equation for $B^2 = \zeta_0$. It follows from here, among others, that the normalizing constant B in Eq. (34) should be taken as equal to 0.470.

Since Eq. (34) and the boundary conditions (35) are independent of any particular velocity distribution U(x, t) along the outer edge of the boundary layer, this equation may be integrated once and for all. In the process of such an integration one determines the dimensionless profiles across transverse sections of the boundary layer, the referred coefficient of friction ζ, and the characteristic quantities H^*, H^{**}, F, T – all as functions of the f_{kn} parameters.

4. We have analyzed Eq. (34) in the local two-parameter approximation, namely in terms of parameters f_{10} and f_{01}. In this version we assumed all parameters except these two to be equal to zero. Furthermore, the derivatives with respect to those parameters were also assumed equal to zero. Thus, the following equation had to be integrated:

$$\frac{\partial^3 \varphi}{\partial \eta^3} + \frac{F + 2f_{10}}{2B^2} \varphi \frac{\partial^2 \varphi}{\partial \eta^2} + \frac{T}{2B^2} \eta \frac{\partial^2 \varphi}{\partial \eta^2} + \frac{f_{10}}{B^2} \left[1 - \left(\frac{\partial \varphi}{\partial \eta} \right)^2 \right] + \frac{f_{01}}{B^2} \left(1 - \frac{\partial \varphi}{\partial \eta} \right) = 0, \tag{36}$$

where

$$F = 2 \left[\zeta - f_{10} (2 + H^*) \right],$$

$$T = -\frac{4 f_{01}}{H^* + 1}, \tag{37}$$

with the boundary conditions:

$$\varphi = \frac{\partial \varphi}{\partial \eta} = 0 \quad \text{at} \quad \eta = 0; \quad \frac{\partial \varphi}{\partial \eta} \to 1 \quad \text{when} \quad \eta \to \infty,$$

$$\varphi = \varphi_0(\eta) \quad \text{at} \quad f_{01} = f_{10} = 0. \tag{38}$$

Unfortunately, the considerable complexity of the nonlinear differential equation (36) makes it impossible to quantitatively estimate the error incurred by the elimination and by the localization of parameters. Therefore, the suitability of any approximation can be assessed only after the solution has been obtained. A comparison of theoretical calculations based on a specifically given velocity distribution along

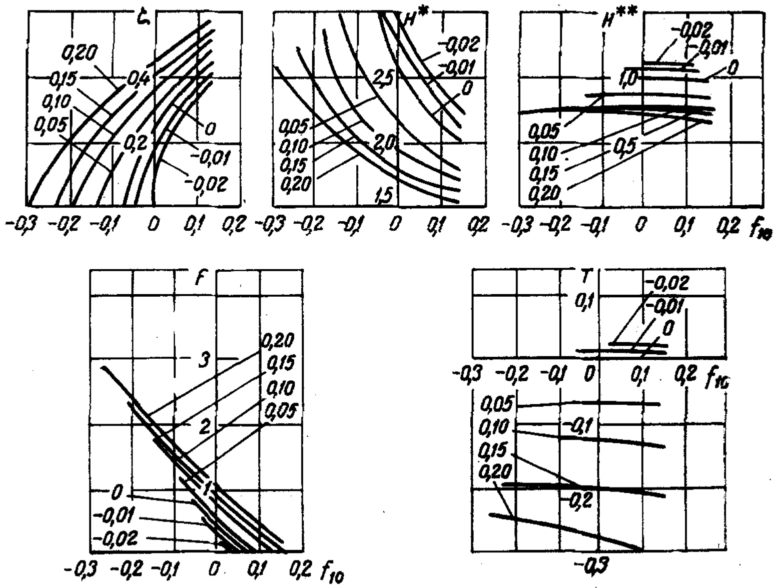

Fig. 1. The referred coefficient of friction and the characteristic quantities as functions of the parameters (the values of f_{01} are indicated next to the curves).

the outer edge of a boundary layer with calculations based on other methods and with experimental data will yield the information concerning the accuracy of the approximation.

Equation (36) has been solved by the trial-and-error method with iterations, using the schedule of differences. The step along variable η was $\Delta\eta = 0.05$ with m = 121 nodes. The step for parameter f_{01} was $\Delta f_{01} = \pm 0.01$ and the step for parameter f_{10} was also $\Delta f_{10} = \pm 0.01$, but the latter step was reduced to one tenth of that as the separation point was being approached.

The essential results of calculations performed on the BÉSM-2 computer are shown graphically in Fig. 1.

We note that functions $\zeta(f_{10})$, $H^*(f_{10})$, $H^{**}(f_{10})$, and $F(f_{10})$ at $f_{01} = 0$ correspond to the solution to the steady-state problem in the local single-parameter approximation. The $\zeta(f_{10}, f_{01})$ graphs show the strong effect of the transiency parameter f_{01} on the friction distribution and, especially, on the position of the separation point in the boundary layer. As this parameter is made larger, the amount of friction increases and the separation point shifts toward larger absolute values of the negative parameter f_{10}. This means that a positive local acceleration draws out the separation point of a boundary layer in the diffuser region. A negative local acceleration facilitates a discontinuity in the stream: separation occurs at a lower absolute value of the negative parameter f_{10}, i.e., in a less diffusory stream than in the case of steady flow. Upon examination of the $\zeta(f_{10})$ curve, we see that with $f_{01} = -0.02$ the stream may separate during a decelerated flow even at a plate, which agrees with the results obtained Struminskii [5].

In order to solve a specific problem with a given velocity distribution U(x, t), one must determine the quantity z(x, t) so that the values of parameters $f_{01}(x, t)$ and $f_{10}(x, t)$ become known and, consequently, also the velocity profiles $\bar{u}(\eta, x, t)$ and the friction $\zeta(x, t)$ in accordance with the solution to the universal equation. In our case we will proceed as follows. We will seek the solution to the momentum equation (17), using the set of velocity profiles and the correspondingly characteristic functions obtained when the universal equation was being integrated. If we disregard the derivatives with respect to the parameters and consider that $f_{01} = (\dot{U}/UU')f_{10}$, then the momentum equation can be rewritten as an equation in the unknown f_{10}:

$$\frac{H^*}{2}\cdot\frac{\partial f_{10}}{\partial t}+\frac{UH^{**}}{2}\cdot\frac{\partial f_{10}}{\partial x}=U'\zeta+f_{10}\left[H^*\left(\frac{\dot{U}'}{2U'}-\frac{\dot{U}}{U}-U'\right)+H^{**}\left(\frac{UU''}{2U'}-2U'\right)\right].$$

Applying this equation to the critical point at the tail end of a circular cylinder, with the velocity $U(x, t) = 2V_0 t \sin(x/r)$ (r is the cylinder radius) at the outer edge of the boundary layer, we have been able to determine approximately the instant when separation begins: $t_S = 0.95\sqrt{r/V_0}$. The coefficient of the square root had in this particular approximation a 5-6% lower value than in the exact solution by Blasius.

NOTATION

x, y	are the longitudinal and transverse coordinate in the boundary layer;
t	is the time;
η	is the dimensionless transverse coordinate;
U	is the velocity at the outer edge of the boundary layer;
ψ	is the flow function;
φ	is the dimensionless flow function;
u, v	are the projections of the velocity in the boundary layer on the x and y axis, respectively;
ρ	is the density of the liquid;
μ, ν	are the dynamic and kinematic viscosity;
h	is the scale function for the transverse coordinate in the boundary layer $z = h^2/\nu$;
F, T, H*, H**	are the characteristic functions;
δ^*	is the displacement thickness;
δ^{**}	is the momentum thickness;
δ_1^{**}	is the energy thickness;
τ_w	is the stress due to surface friction;
ζ	is the referred coefficient of friction;
e	is the dissipation function;
B	is the normalizing factor;
f_{kn}	is the dimensionless parameters;
r	is the cylinder radius.

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